Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Specimen Paper

(Time: 1 hour 30 minutes) | Paper Reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1. The point $P(7, -2)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of P under the transformation represented by the curve with equation

(a) $y = f(x + 4) + 1$

(2)

(b) $y = f(-2x) + 7$

(2)

1.a) $P : (7, -2)$

$y = f(x + 4) + 1$

translation through the vector $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$\therefore P(7, -2) \rightarrow (7-4, -2) \rightarrow (3, -2+1)$

$= (3, -1)$

b) $y = f(-2x) + 7$

translation $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$

stretch : scale factor $-\frac{1}{2}$

parallel to the x-axis

$\therefore P(7, -2) \rightarrow (7 \times -\frac{1}{2}, -2) \rightarrow (-\frac{7}{2}, -2+7)$

$\rightarrow (-\frac{7}{2}, 5)$

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2. (a) Using the identity for $\cos(A \pm B)$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A$$

(2)

(b) Hence find, using calculus, the exact value of

$$\int_0^{\frac{\pi}{4}} 3\sin^2 2x \, dx$$

(3)

2.a) USING COMPOUND ANGLE FORMULAE → $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$\cos(2A) = \cos(A + A)$

$= \cos(A)\cos(A) - \sin(A)\sin(A)$

$= \cos^2(A) - \sin^2(A)$ ← $\cos^2(A) + \sin^2(A) = 1$

$= (1 - \sin^2(A)) - \sin^2(A)$

$= 1 - 2\sin^2(A)$

b) $\int_0^{\pi/4} 3\sin^2(2x) \, dx$

$= \frac{3}{2}\int_0^{\pi/4} 2\sin^2(2x) \, dx$ ← using identity from (a)

$= \frac{3}{2}\int_0^{\pi/4} 1 - \cos(4x) \, dx$

$= \frac{3}{2}\left[x - \frac{1}{4}\sin(4x)\right]_0^{\pi/4}$

$= \frac{3}{2}\left(\frac{\pi}{4}\right) = \frac{3\pi}{8}$

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3. Guinea pigs and rabbits were introduced onto an island at the same time.

The number of guinea pigs, G, t months after they were introduced onto the island is modelled by the equation

$$G = a + 60e^{-0.05t}$$

where a is a positive constant.

The number of rabbits, R, t months after they were introduced onto the island is modelled by the equation

$$R = 100 + 80e^{0.05t}$$

Given that there were twice as many guinea pigs as rabbits introduced onto the island

(a) find the value of a.
(2)

When $t = T$, the number of rabbits on the island is equal to the number of guinea pigs on the island.

Using these models,

(b) find the value of T, giving your answer to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

3. a) $G_0 = a + 60e^{-0.05(0)} = a + 60$

$R_0 = 100 + 80e^{0.05t} = 100 + 80 = 180$

given $G_0 = 2R_0$

$a + 60 = 2(180)$

$a = 300$

b) when $t = T$, $G_T = R_T$

$300 + 60e^{-0.05T} = 100 + 80e^{0.05T}$

$80e^{0.05T} - 200 - 60e^{-0.05T} = 0$ ← multiply through by $e^{0.05T}$

$80e^{0.1T} - 200e^{0.05T} - 60 = 0$

Question 3 continued

let $e^{0.05T} = u$

$80u^2 - 200u - 60 = 0$

$u = e^{0.05T} = \frac{5 \pm \sqrt{37}}{4}$

reject negative as e^A $A \in \mathbb{R}$ is always > 0

$\therefore T = 20 \ln\left(\frac{5+\sqrt{37}}{4}\right)$

$= 20.4$

Q3

(Total for Question 3 is 6 marks)

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In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

4. Solve, for $0 \leqslant \theta < 2\pi$, the equation

$$3\sin 2\theta + 5\cos 2\theta = 4$$

giving each answer to 2 significant figures.

(7)

4. $3\sin(2\theta) + 5\cos(2\theta) = 4 \qquad 0 \leq \theta < 2\pi$

compare to $R\sin(2\theta + \alpha)$

using compound angle formulae

$\sin(A+B)$

$= \sin(A)\cos(B) + \sin(B)\cos(A)$

$R(\sin 2\theta \cos\alpha + \sin\alpha \cos 2\theta) = 3\sin(2\theta) + 5\cos(2\theta)$

Compare expanded expressions

$R\sin 2\theta \cos\alpha + R\sin\alpha \cos 2\theta = 3\sin(2\theta) + 5\cos(2\theta)$

$R\cos\alpha = 3 \qquad R\sin\alpha = 5$

$\frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \tan\alpha = \frac{5}{3}$

$\alpha = 1.03$

$(R\cos\alpha)^2 + (R\sin\alpha)^2$

$\cos^2 A + \sin^2 A = 1$ identity

$= R^2(\cos^2\alpha + \sin^2\alpha) = R^2(1)$

$= 3^2 + 5^2$

$= 34 \qquad \therefore R^2 = 34$

$R = \sqrt{34}$

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Question 4 continued

$\therefore \quad 3\sin(2\theta) + 5\cos(2\theta)$

$= \sqrt{34}\ \sin(2\theta + 1.03) = 4$

$\sin(2\theta + 1.03) = \dfrac{4}{\sqrt{34}}$

$2\theta + 1.03 = 0.756 \cup 2.386 \cup 7.039 \cup 8.669 \cup 13.322 \cup 14.95 \cup \ldots$

$\theta = \cancel{-0.14} \cup 0.68 \cup 3.0 \cup 3.8 \cup 6.1 \cup \cancel{7.0}$

reject these 2 solutions
as $0 \leq \theta < 2\pi$

$\therefore \quad \theta = 0.68, 3.0, 3.8, 6.1$

(Total for Question 4 is 7 marks)

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Q4

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5. A function f is defined by

$$f(x) = \frac{7}{x+3} - \frac{5x+22}{x^2+7x+12} \qquad x \in \mathbb{R}, x > -3$$

(a) Show that f(x) can be written in the form $\frac{A}{Bx+C}$ where A, B and C are constants to be found.

(3)

(b) Hence find f^{-1}

(3)

(c) Solve $ff(x) = \frac{2}{5}$

(3)

5. a) $f(x) = \frac{7}{x+3} - \frac{5x+22}{x^2+7x+12}$ $\quad x \in R$, $x > -3$

$= \frac{7}{x+3} - \frac{5x+22}{(x+3)(x+4)}$

$= \frac{7(x+4) - 5x - 22}{(x+3)(x+4)}$

$= \frac{7x+28-5x-22}{(x+3)(x+4)}$

$= \frac{2x+6}{(x+3)(x+4)}$

$= \frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x+4)}$

$= \frac{2}{x+4}$

$A = 2$
$B = 1$
$C = 4$

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Question 5 continued

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b) to find $f^{-1}(x)$: $f(x) = \frac{2}{x+4}$

① write the function using a "y" and set equal to "x" $x = \frac{2}{y+4}$

② rearrange to make y the subject : $xy + 4x = 2$

$y = \frac{2-4x}{x}$

③ replace y with $f^{-1}(x)$:

$\therefore f^{-1}(x) = \frac{2-4x}{x} = \frac{2}{x} - 4$

Because we are told to find $f^{-1}(x)$, we must also state the domain of the inverse function :

↑ domain refers to the set of values we are allowed to plug into our function

domain of inverse function = range of function

↑ range refers to all possible values of a function

$\therefore$ domain of $f^{-1}(x)$ = range of $f(x)$

$f(x) = \frac{2}{x+4}$ $x > -3$, $x \in \mathbb{R}$ → when $x = -3$ → $f(x) = 2$

→ as $x \to \infty$ → $f(x) \to 0$

$\therefore$ range of $f(x)$ is $0 < f(x) < 2$

$\therefore f^{-1}(x) = \frac{2}{x} - 4$ $0 < x < 2$, $x \in \mathbb{R}$

Q5

(Total for Question 5 is 9 marks)

Turn over

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Question 5 continued

c) $f(f(x))$

$= f\left(\frac{2}{x+4}\right)$

$= \dfrac{2}{\frac{2}{x+4} + 4}$

$= \dfrac{2(x+4)}{2+4x+16}$

$= \dfrac{2x+8}{4x+18}$

$= \dfrac{x+4}{2x+9}$

given $f(f(x)) = \frac{2}{5}$

$\dfrac{x+4}{2x+9} = \dfrac{2}{5}$

$5x+20 = 4x+18$

$x = -2$

Q5

(Total for Question 5 is 9 marks)

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6.

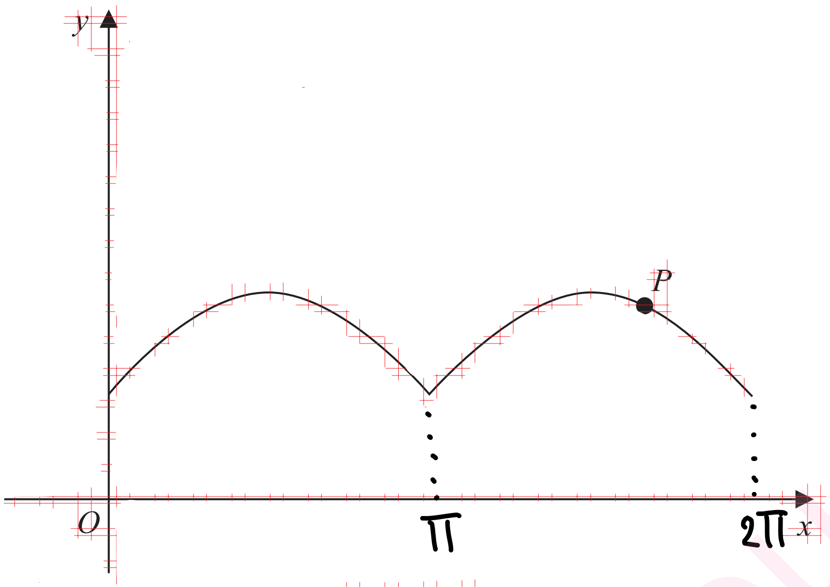

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = |\sin x| + 1 \qquad 0 \leqslant x \leqslant 2\pi$$

The point $P(a, b)$ lies on the curve and is shown on Figure 1.

Given that the gradient of the curve at P is $-\frac{1}{2}$

(a) find the exact value of a and the exact value of b.

(4)

A straight line with **positive** gradient passes through P.

Given that the straight line intersects the curve at exactly three distinct points,

(b) find the range in values of the gradient of the line.

(3)

6. a) $y = |\sin(x)| + 1 \qquad 0 \leq x \leq 2\pi$

$P(a, b) \rightarrow$ gradient at $P = \frac{dy}{dx}(P) = -\frac{1}{2}$

Because P is after π & $\therefore$ $\sin(a)$ is negative

↳ at P / after π $\rightarrow y = -\sin(x) + 1$

$\therefore \frac{dy}{dx} = -\cos(x)$

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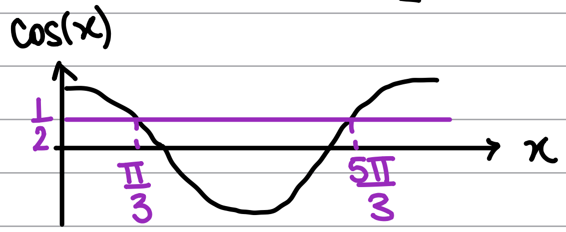
Question 6 continued

$$\text{gradient at } P = -\frac{1}{2} = \frac{dy}{dx}(P) = -\cos(a)$$

$$\cancel{-}\cos(a) = \cancel{-}\frac{1}{2}$$



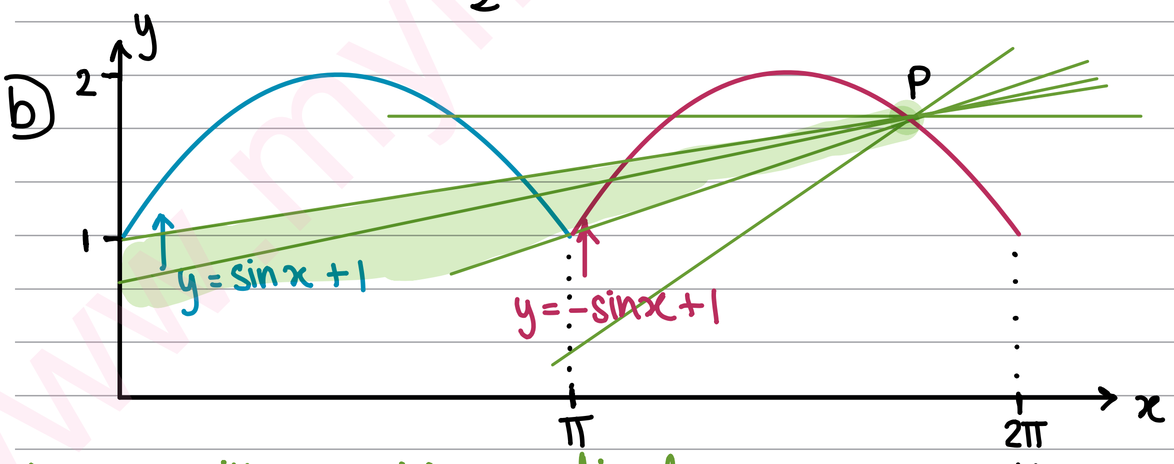
$$\therefore \quad a = \cancel{\frac{\pi}{3}} \cup \frac{5\pi}{3}$$

reject as $\frac{\pi}{3}$ is $< \pi$ & we can see from the graph that a is definitely $> \pi$

$$\therefore \ a = \frac{5\pi}{3}, \quad b = \left|\sin\left(\frac{5\pi}{3}\right)\right| + 1$$

$$= \frac{\sqrt{3}}{2} + 1$$

b)



A line with a positive gradient only passes through P & 2 other points when gradient (m) lies between certain values

Q6

(Total for Question 6 is 7 marks)

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Question 6 continued

* when line passes $(\pi, 1)$, there are only 2 intersections

at this point $m = \dfrac{\left(\frac{\sqrt{3}}{2}+1\right) - 1}{\frac{5\pi}{3} - \pi} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{2\pi}{3}} = \dfrac{3\sqrt{3}}{4\pi}$

* when the line is less steep, i.e. $m < \dfrac{3\sqrt{3}}{4\pi}$

there are 3 intersections

* however if the line becomes so shallow that it goes through $(0,1)$, there are more than 3 intersections

$m = \dfrac{\left(\frac{\sqrt{3}}{2}+1\right) - 1}{\frac{5\pi}{3} - 0} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{5\pi}{3}} = \dfrac{3\sqrt{3}}{10\pi}$

* once the line is shallower than this there are more than 3 intersections (as line intersects $y = \sin x + 1$ more than once)

$\therefore$ 3 intersections : $\dfrac{3\sqrt{3}}{10\pi} < m < \dfrac{3\sqrt{3}}{4\pi}$

Q6

(Total for Question 6 is 7 marks)

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7. A curve has equation

$$y = e^{2\sqrt{3}x} \cos 2x \qquad 0 < x < \pi$$

(a) Find $\frac{dy}{dx}$ **(2)**

(b) Hence, using algebra and showing your working, find the exact coordinates of the stationary points of the curve.

(Solutions based entirely on graphical or numerical methods are not acceptable.) **(6)**

7. a) $y = e^{2\sqrt{3}x} \cos(2x) \qquad 0 < x < \pi$

$\boxed{\text{PRODUCT RULE : } y = uv \quad y' = u'v + uv'}$

$u = e^{2\sqrt{3}x} \qquad \frac{du}{dx} = 2\sqrt{3}e^{2\sqrt{3}x}$

$v = \cos(2x) \qquad \frac{dv}{dx} = -2\sin(2x)$

$$\frac{dy}{dx} = \left(2\sqrt{3}e^{2\sqrt{3}x}\right)\left(\cos(2x)\right) + \left(e^{2\sqrt{3}x}\right)\left(-2\sin(2x)\right)$$

$$= e^{2\sqrt{3}x}\left(2\sqrt{3}\cos(2x) - 2\sin(2x)\right)$$

b) At stationary points $\rightarrow \frac{dy}{dx} = 0$

$$e^{2\sqrt{3}x}\left(2\sqrt{3}\cos(2x) - 2\sin(2x)\right) = 0$$

$$2\sqrt{3}\cos(2x) = 2\sin(2x)$$

$$\tan(2x) = \frac{\cancel{2}\sqrt{3}}{\cancel{2}} = \sqrt{3}$$

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Question 7 continued

$$x = \frac{\pi}{6} \cup \frac{2\pi}{3}$$

$$y = e^{2\sqrt{3}\left(\frac{\pi}{6}\right)} \cos\left(\frac{2\pi}{6}\right) \qquad y = e^{2\sqrt{3}\left(\frac{2\pi}{3}\right)} \cos\left(2 \times \frac{2\pi}{3}\right)$$

$$= \frac{e^{\frac{\sqrt{3}\pi}{3}}}{2} \qquad = -\frac{e^{\frac{4\sqrt{3}}{3}\pi}}{2}$$

$\therefore$ Stationary points : $\left(\frac{\pi}{6}, \frac{e^{\frac{\sqrt{3}\pi}{3}}}{2}\right)$ & $\left(\frac{2\pi}{3}, -\frac{e^{\frac{4\sqrt{3}\pi}{3}}}{2}\right)$

Q7

(Total for Question 7 is 8 marks)

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8. Liam monitored the population of a small country over a 10-year period.

The population, P, measured in thousands of people, is modelled by the equation

$$P = ab^{-t}$$

where a and b are constants and t is the number of years since monitoring began.

(a) Show that this equation can be expressed in the form

$$\log_{10} P = \log_{10} a - t \log_{10} b$$

(1)

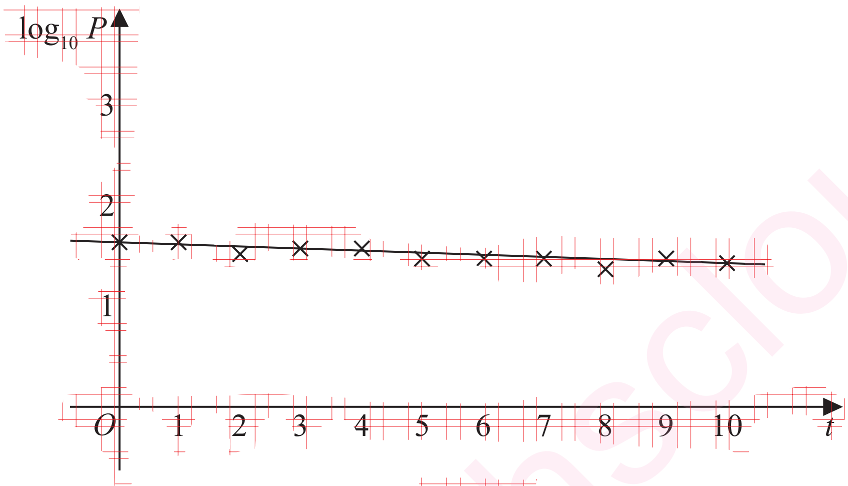

Figure 2

Figure 2 shows a line of best fit for values of t and $\log_{10} P$

The line of best fit passes through points (0, 1.6) and (10, 1.4)

Using this information,

(b) find the value of a and the value of b, giving each answer to 4 significant figures.

(4)

Hence, according to the model,

(c) find the rate at which the population was changing exactly 8 years after monitoring began.

(3)

8.a) $P = ab^{-t}$

$\log_{10}(P) = \log_{10}(ab^{-t})$

$\log_{10}(P) = \log_{10}(a) + \log_{10}(b^{-t})$

$\log_{10}(P) = \log_{10}(a) - t\log_{10}(b)$

LOG RULES

$a\log_b(c) = \log_b(c^a)$

$\log_a b + \log_a c = \log_a(bc)$

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Question 8 continued

b) $\log_{10}(P) = \log_{10}(a) - t\log_{10}(b)$

$y = c + x\, m$

given that lobf passes through $(0, 1.6)$ & $(10, 1.4)$

$$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{1.4 - 1.6}{10 - 0} = -0.02 = m$$

$$\cancel{-}0.02 = \cancel{-}\log_{10}(b)$$

$$b = 10^{0.02} = 1.047$$

plug in 1 known point :

$$1.6 = \log_{10}(a) - \cancel{(0)\log_{10}(b)}$$

$$\therefore a = 10^{1.6} = 39.81$$

c) rate of change of pop $= \dfrac{dP}{dt}$

$$P = ab^{-t}$$

$y = a^x$ ← rewrite a^x in terms of e

$a = e^{\ln(a)}$

$\frac{d}{dx}(e^{kx}) = ke^{kx}$

$\therefore y = (e^{\ln(a)})^x \rightarrow \dfrac{dy}{dx} = \ln(a) \times e^{(\ln a)x} = \ln(a) \times a^x$

Turn over

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Question 8 continued

$\longrightarrow \quad \frac{dP}{dt} = -a\ln(b)\,b^{-t}$

At 8 years $\rightarrow \frac{dP}{dt} = -39.81 \times \ln(1.047) \times 1.047^{-8}$

$= -1.27$

$\therefore$ a decrease of 1270 people per year

Q8

(Total for Question 8 is 8 marks)

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9. A curve has equation

$$x = \frac{\sin y - \cos y}{\cos y + \sin y} \qquad -\frac{\pi}{4} < y < \frac{3\pi}{4}$$

(a) Show that the equation of the curve can be written as

$$x = \frac{-\cos 2y}{1 + \sin 2y} \qquad -\frac{\pi}{4} < y < \frac{3\pi}{4}$$

(3)

(b) Hence, or otherwise, show that

$$\frac{dx}{dy} = \frac{2}{1 + \sin 2y}$$

(2)

A point $P(x, y)$ lies on the curve.

Given that at P

- $\frac{dy}{dx} = \frac{1}{4}$
- $y < 0$

(c) find the exact coordinates of P

(5)

9.a) $x = \dfrac{\sin(y) - \cos(y)}{\cos(y) + \sin(y)}$

$x = \dfrac{(\sin(y) - \cos(y))(\sin(y) + \cos(y))}{(\cos(y) + \sin(y))(\cos(y) + \sin(y))}$

$= \dfrac{\sin^2(y) - \cos^2(y)}{\cos^2(y) + \sin^2(y) + 2\sin(y)\cos(y)}$

$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$

$\boxed{\cos^2(A) + \sin^2(A) = 1}$

$\boxed{\cos(2A) = \cos^2(A) - \sin^2(A)}$

$= \dfrac{-\cos(2y)}{1 + \sin(2y)}$

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Question 9 continued

b) $x = \dfrac{-\cos(2y)}{1+\sin(2y)}$

Quotient rule for differentiating : $y = \dfrac{u}{v} \rightarrow \dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = -\cos(2y)$ $\qquad \dfrac{du}{dx} = 2\sin(2y)$

$v = 1+\sin(2y)$ $\qquad \dfrac{dv}{dx} = 2\cos(2y)$

$$\frac{dx}{dy} = \frac{(1+\sin(2y))(2\sin(2y)) - (-\cos(2y))(2\cos(2y))}{(1+\sin(2y))^2}$$

$$= \frac{2\sin(2y) + 2\sin^2(2y) + 2\cos^2(2y)}{(1+\sin(2y))^2}$$

$$= \frac{2\cancel{(\sin(2y)+1)}}{\cancel{(1+\sin(2y))^2}}$$

$$= \frac{2}{1+\sin(2y)}$$

c) $P : (x, y)$

$\dfrac{dx}{dy} = \dfrac{2}{1+\sin(2y)} = 4$ $\qquad x = \dfrac{-\cos(2y)}{1+\sin(2y)}$

$\dfrac{1}{2} = 1+\sin(2y)$ $\qquad \downarrow$

$\qquad x = -\sqrt{3}$

$\sin(2y) = -\dfrac{1}{2} \rightarrow -\dfrac{\pi}{12}$

$\therefore P\left(\sqrt{3}, -\dfrac{\pi}{12}\right)$

10.

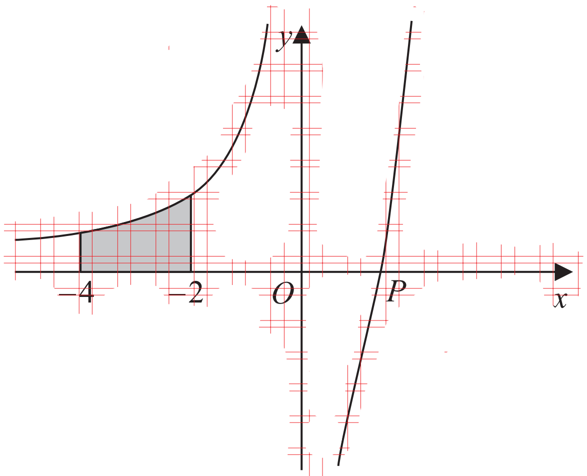

Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = e^{2x-3} - \frac{4}{3x} \qquad x \in \mathbb{R}, x \neq 0$$

The curve crosses the x-axis at the point $P(\alpha, 0)$.

(a) Show that

$$\alpha = \frac{1}{2}\left(\ln\left(\frac{4}{3\alpha}\right) + 3\right)$$

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2}\left(\ln\left(\frac{4}{3x_n}\right) + 3\right)$$

is used to find an approximation to α.

(b) Taking $x_0 = 2$ find the value of x_1 and the value of x_5

Give each answer to 4 decimal places.

(2)

(c) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places α is 1.456

(2)

The finite region, shown shaded in Figure 3, is bounded by the curve, the line with equation $x = -4$, the x-axis and the line with equation $x = -2$

(d) Using integration find, in simplest form, the exact area of the shaded region.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

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Question 10 continued

10. (a) $y = e^{2x-3} - \frac{4}{3x}$ $\qquad x \in \mathbb{R}$, $x \neq 0$

$P: (\alpha, 0)$

$$0 = e^{2\alpha - 3} - \frac{4}{3\alpha}$$

$$\frac{4}{3\alpha} = e^{2\alpha - 3}$$

$$2\alpha + 3 = \ln\left(\frac{4}{3\alpha}\right)$$

$$\alpha = \frac{1}{2}\left(\ln\left(\frac{4}{3\alpha}\right) - 3\right)$$

(b) $x_{n+1} = \frac{1}{2}\left(\ln\left(\frac{4}{3x_n}\right) + 3\right)$

$x_0 = 2$

$x_1 = x_{0+1} = \frac{1}{2}\left(\ln\left(\frac{4}{3(2)}\right) + 3\right) = 1.2973$

$x_2 = 1.5137$

$x_3 = 1.4366$

$x_4 = 1.4627$

$x_5 = 1.4537$

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Question 10 continued

c) $f(x) = e^{2x-3} - \frac{4}{3x}$

To show $\alpha = 1.456$ to 3 d.p., we must check the values of $f(1.4565)$ & $f(1.4555)$

$f(1.4555) = -0.0012199$

$f(1.4565) = 0.0012405$

because P is continuous between 1.4555 & 1.4565

the sign change shows that P crosses the axis between 1.4555 & 1.4565 & so there is a root.

$\therefore \alpha = 1.456$ to 3 d.p.

d) $\int_{-4}^{-2} e^{2x-3} - \frac{4}{3x} \, dx$

$= \int_{-4}^{-2} e^{-3}(e^{2x}) - \frac{4}{3}x^{-1} \, dx$

$= \left[\frac{1}{2}e^{-3}(e^{2x}) - \frac{4}{3}\ln|x|\right]_{-4}^{-2}$

$= \left[\frac{1}{2}e^{2x-3} - \frac{4}{3}\ln|x|\right]_{-4}^{-2}$

Turn over

S 6 4 6 1 5 A 0 2 7 2 8

Question 10 continued

$$= \frac{1}{2}e^{2(-2)-3} - \frac{4}{3}\ln|-2| - \frac{1}{2}e^{2(-4)-3} + \frac{4}{3}\ln|-4|$$

$$= \frac{e^{-7}}{2} - \frac{e^{-11}}{2} + \frac{4}{3}(\ln 4 - \ln 2)$$

$$= \frac{e^{-7}}{2} - \frac{e^{-11}}{2} + \frac{4}{3}\ln\left(\frac{4}{2}\right)$$

LOG RULES

$$\boxed{\log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)}$$

$$= \frac{e^{-7}}{2} - \frac{e^{-11}}{2} + \frac{4}{3}\ln(2)$$

Leave blank

Q10

(Total for Question 10 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS